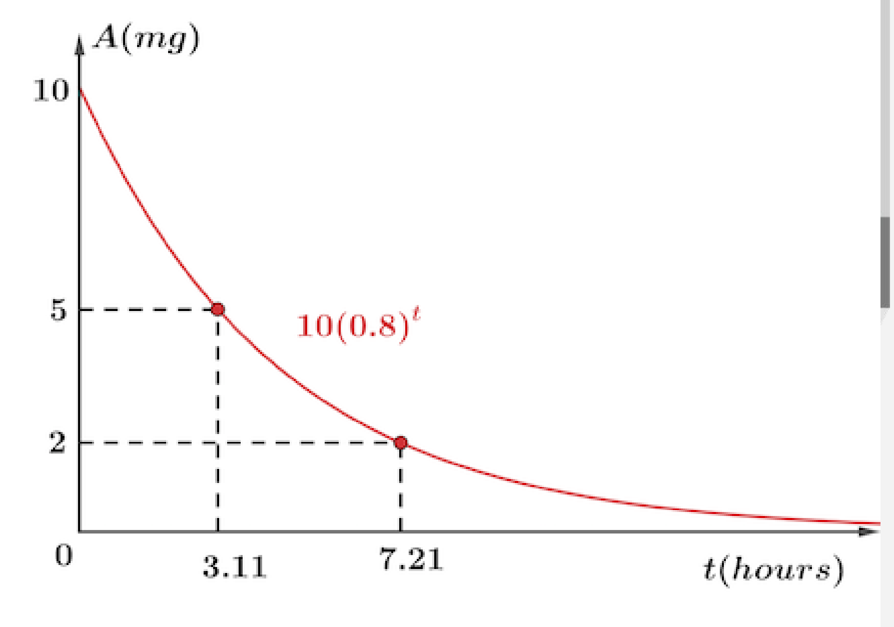
The Maths Behind Drug Dosage

Exponential Decay and Half lives in Medicine

What happens when someone takes a drug?

- the amount of drug in the bloodstream rises
- then gradually falls
- it does not decrease by the same amount every hour
- it decreases by the same percentage
- doctors must avoid overdose or ineffective doses



The Differential Equation

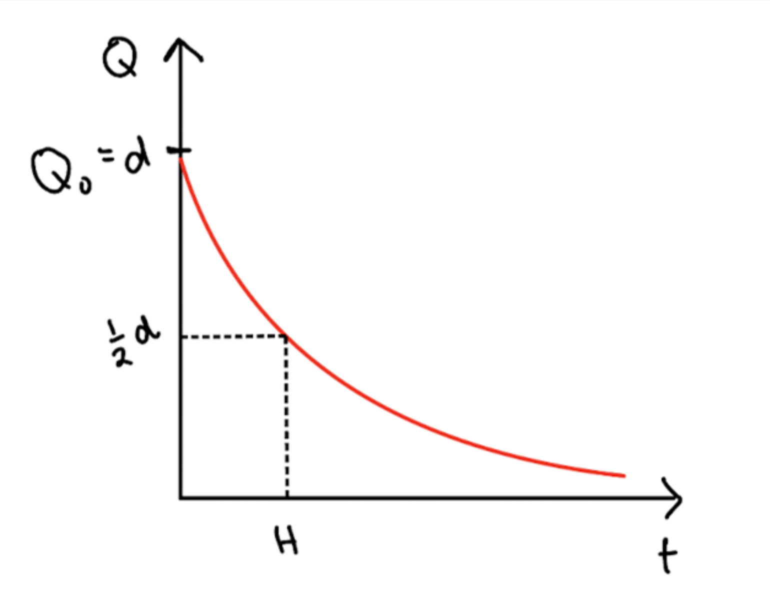
KEY ASSUMPTION: "the rate at which the drug leaves the body is proportional to the amount currently present"

$$dQ/dt = -kQ$$

($k>0$ so $-k<0$)

The Exponential Function

$$Q(t) = Q_0 e^{-kt}$$



The Variables Used in the Model

(d) = dosage amount of drug in mg

(Q(t)) = quantity of drug (mg) in the bloodstream at time (t)

(t) = time (hours)

(H) = half-life (hours)

(k) = decay constant

(Q_0) = initial dose (mg)

<u>Assumptions:</u>

- the drug enters the bloodstream instantly (through IV)
- the body behaves as one compartment
- the rate of removal depends on how much drug is present

Example

$Q(t)=500e^{-0.2t}$

Question:

How much drug remains after **5** hours?

Substitute t=5:

$Q(5)=500e^{-0.2(5)}$

$Q(5)=500e^{-1}$

$Q(5)\approx 184$ mg

Half Lives

$$Q(t) = Q_0 e^{-kt}$$

$$Q = \frac{Q_0}{2}$$

$$\frac{Q_0}{2} = Q_0 e^{-kH}$$

$$\frac{1}{2} = e^{-kH}$$

$$-\ln(2) = -k(H)$$

$$H = \frac{\ln(2)}{k} \quad \text{or} \quad k = \frac{\ln(2)}{H}$$

Reversing It - Natural Logarithms

The exponential model:

$Q(t) = Q_0e^{-kt}$

- lets us calculate the amount of drug remaining after a certain time
- What if I wanted to find out how long until “x” mg of drug is left?
- time is trapped in the exponent, we use natural logarithms to solve for it.

Example

Suppose:
$Q(t) = 500e^{-0.2t}$

When will only **50 mg remain**?

Substitute:
$Q = 50$
$Q_0 = 500$

This gives:

$50 = 500e^{-0.2t}$

Divide both sides by 500:

$0.1 = e^{-0.2t}$

$\ln(0.1) = -0.2t$

Then divide by -0.2:

$t \approx 11.5$ hrs

150 Word Conclusion

Mathematical modelling provides a useful way to understand and predict how drugs behave in the body over time. By assuming that the rate at which a drug is eliminated is proportional to the amount currently present, we can form the differential equation $dt/dQ= -kQ$. Solving this equation leads to an exponential decay model, which simply describes how the concentration of drugs decreases in the bloodstream. Using this model, we can calculate quantities such as the half life of the drug, safe and effective dosage intervals, and calculating the time it takes for x amount of drug to eliminate. Natural logarithms play an important role because it allows us to reverse exponential relationships and solve for t when a concentration is known. Although real pharmacokinetic models can be more complex and may involve multiple components within the body, along with different methods of taking the drug (this model is exclusively for drugs given through IV) the exponential model provides a simple mathematical approximation. Overall, this topic demonstrates how differentiation, exponential functions, and logarithms can be applied to solve real world medical problems that doctors and pharmacologists can use to approximate drug dosages.