The Birthday Paradox

How many people need to be in a room before the probability 2 of them share a birthday surpasses 50%?

It's 23 people - less than we might think - but why?

JANUARY

SUNDAY	MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY
[redacted]						

FEBRUARY

SUNDAY	MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY
[redacted]						

MARCH

SUNDAY	MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY
			[redacted]			

APRIL

SUNDAY	MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY
[redacted]						
					[redacted]	

MAY

SUNDAY	MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY
[redacted]						

JUNE

SUNDAY	MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY
[redacted]						
			[redacted]			

JULY

SUNDAY	MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY
[redacted]						

AUGUST

SUNDAY	MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY
[redacted]						

SEPTEMBER

SUNDAY	MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY
[redacted]						
				[redacted]		

OCTOBER

SUNDAY	MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY
[redacted]						

NOVEMBER

SUNDAY	MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY
		[redacted]				

DECEMBER

SUNDAY	MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY
[redacted]						
					[redacted]	

If we assume there are no twins, no leap years and the probability of each birthday is the same…

There are only 2 possible outcomes here: either 2 people share a birthday, or they don’t.

Therefore:

Probability 2 people share a birthday + probability they don’t = 1

P(A) = 1 - P(not A) where A is a particular outcome

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			[redacted]			

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				[redacted]		

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Number of possible birthday dates each person could have if no two people shared

$$\frac{365}{365} \times \frac{364}{365} \times \frac{363}{365} \times \frac{362}{365} \times \frac{361}{365} \times \frac{360}{365} \times \frac{359}{365} \ldots\ldots \frac{343}{365} = 0.4927$$

Number of total possible birthday dates (days in a year)

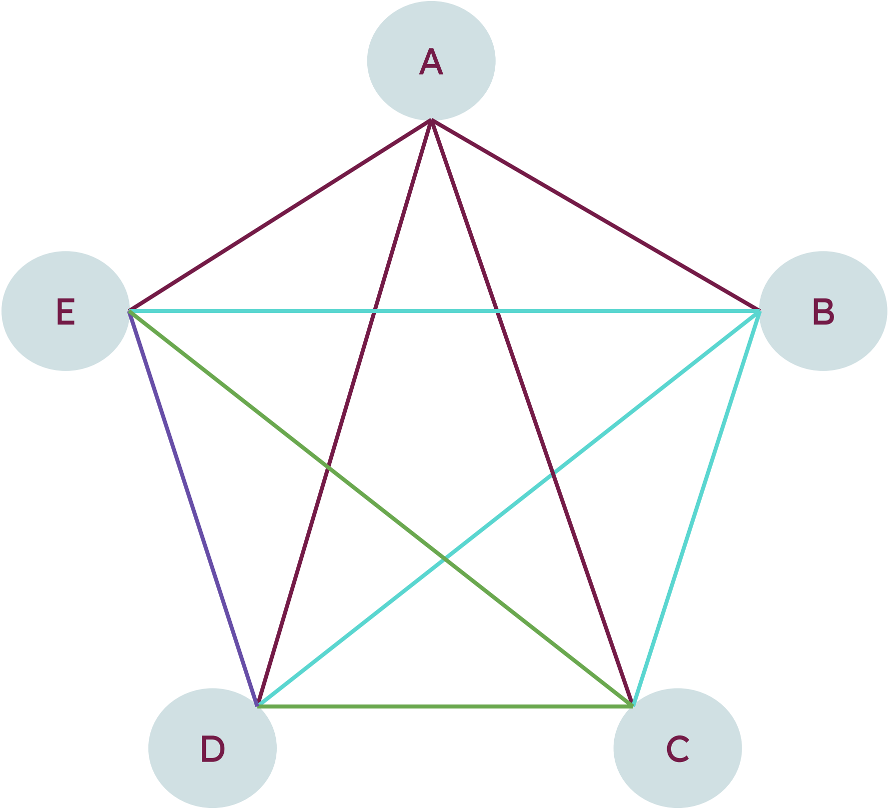
The 23rd person

$$1 - 0.4927 = 0.5073$$

So probability 2 people share a birthday = 50.73%

Combinatorics

A branch of maths concerning counting - i.e the number of ways of choosing some objects from a collection or the number of ways of arranging them



Or as a list:

AB (BA)
AC (CA)
AD (DA)
AE (EA)
BC (CB)
BD (DB)
BE (EB)
CD (DC)
CE (EC)
DE (ED)

$$\frac{5 \times 4}{2} = 10$$

$$\text{Pairs} = \frac{n(n-1)}{2}$$

But why?

As the number of people increases, the number of possible pairings increases quadratically - much more than we realise.

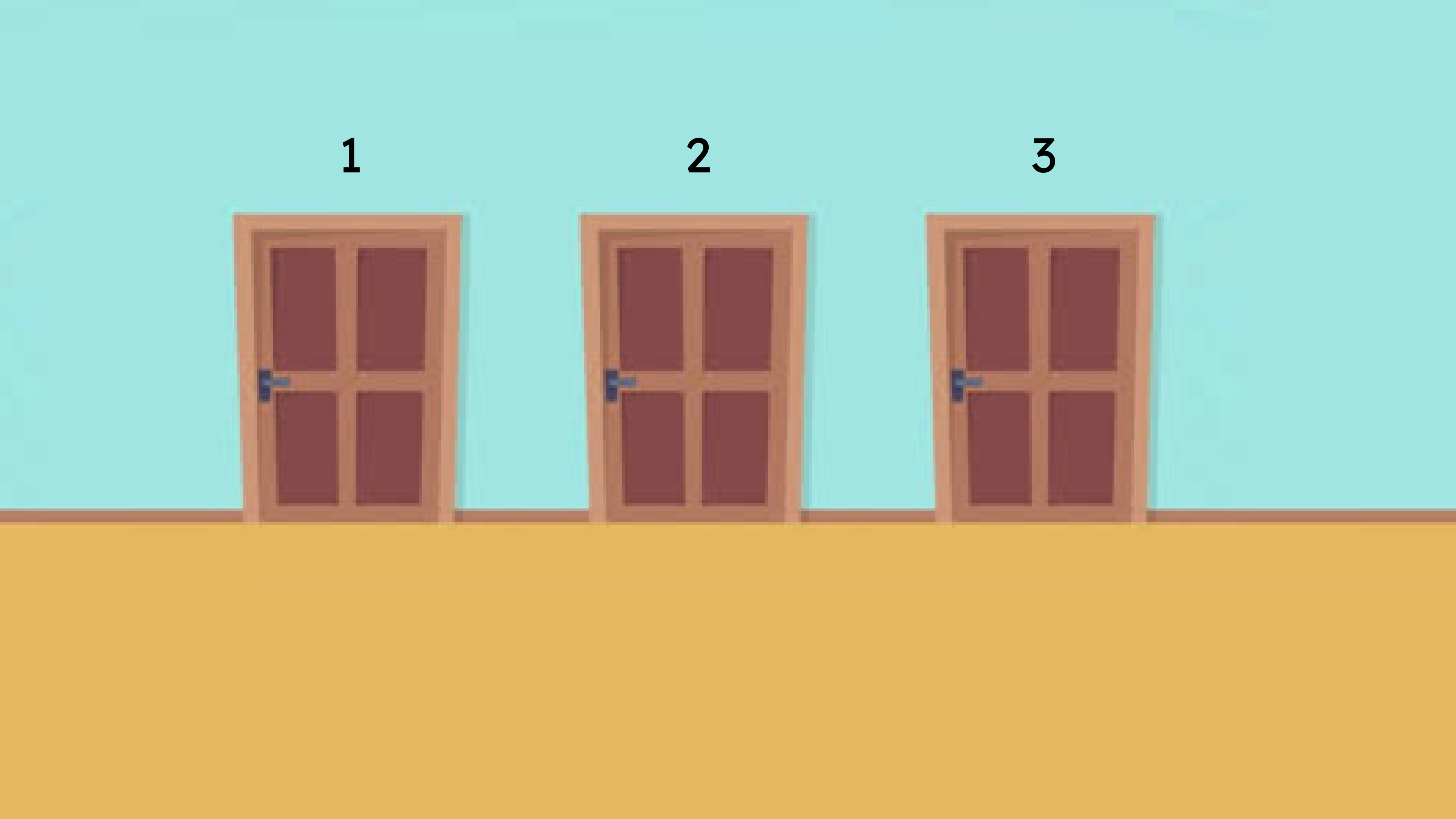
It’s much easier for us to approach this problem from a linear perspective than a quadratic one, so we overestimate the answer.

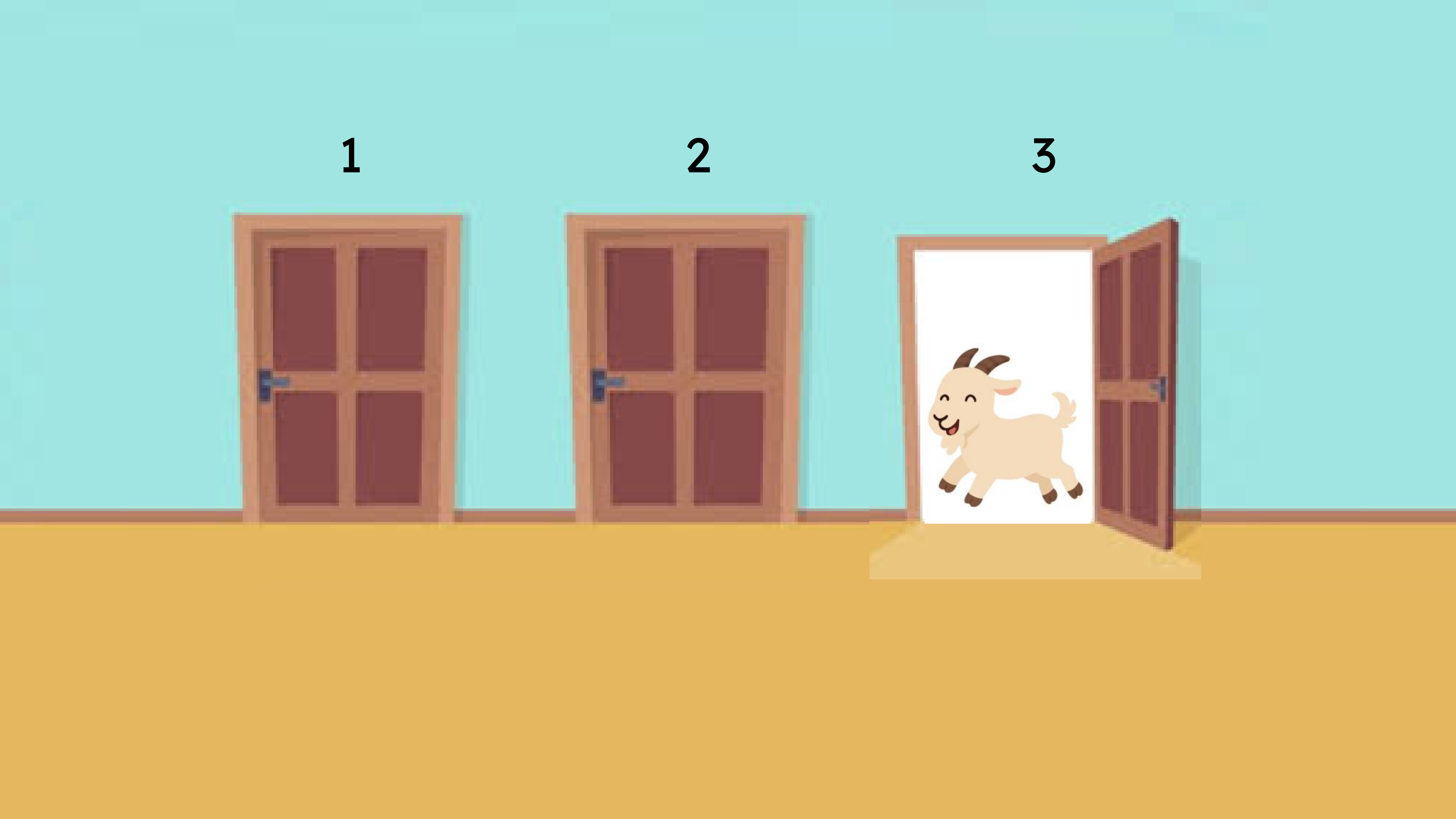
$$\text{Number of pairs} = \frac{n(n-1)}{2}$$

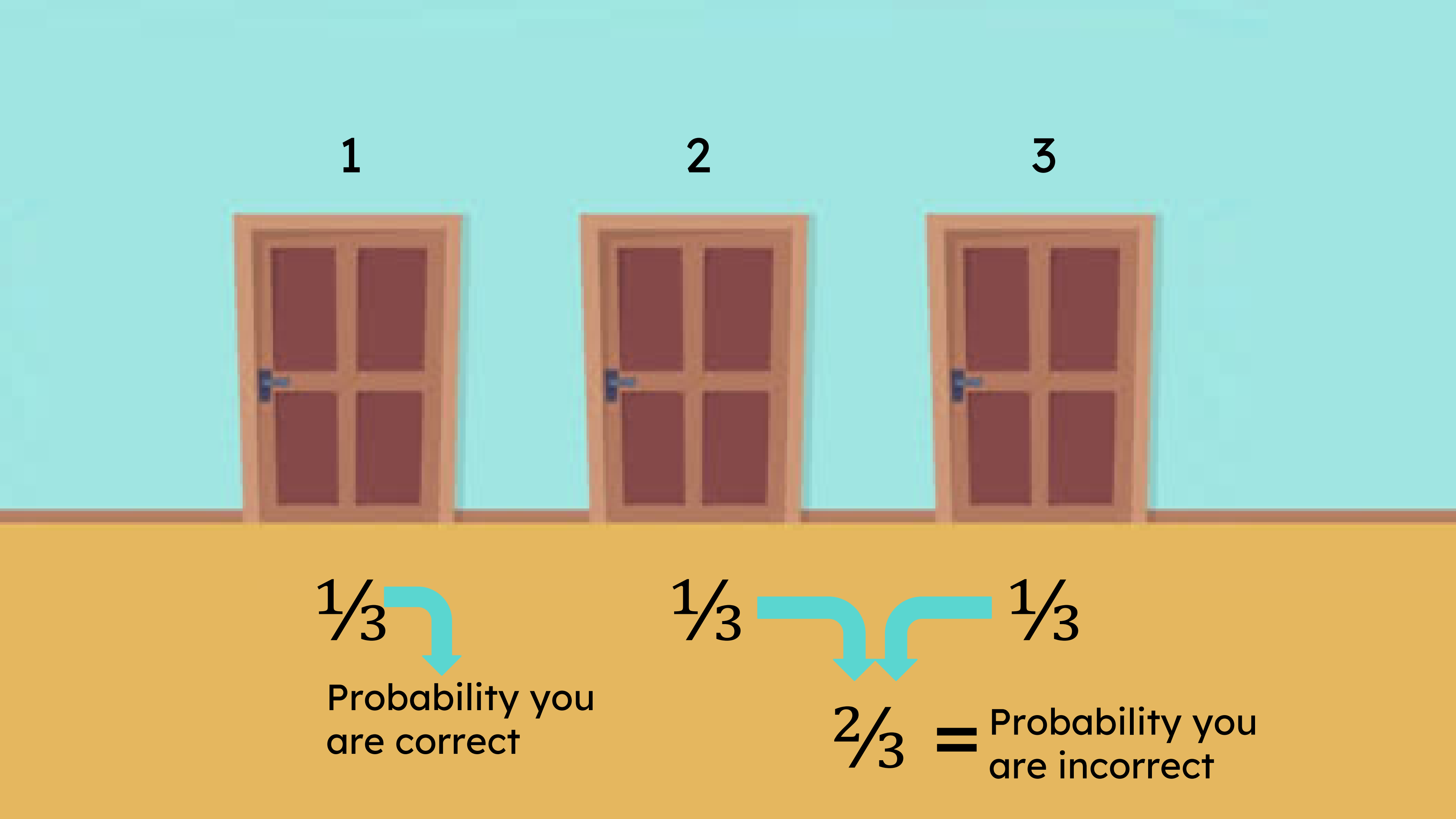
where n is the number of people

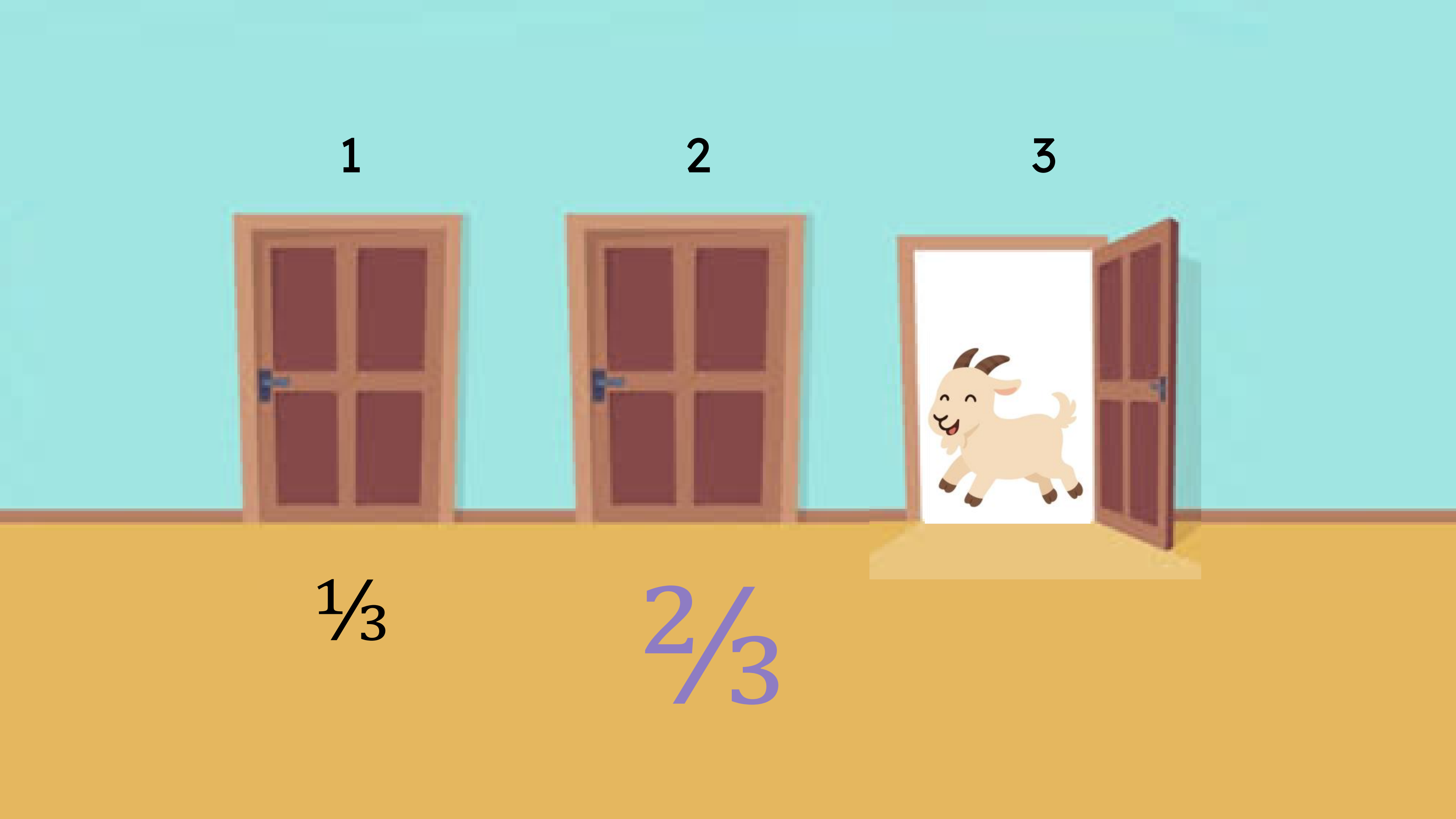
The Monty Hall Problem

Veridical paradox - a situation that seems counterintuitive or even impossible but makes logical sense once you look deeper into it.


1
2
3


1
2
3


1
2
3
1/3
1/3
1/3
Probability you are correct
2/3 = Probability you are incorrect

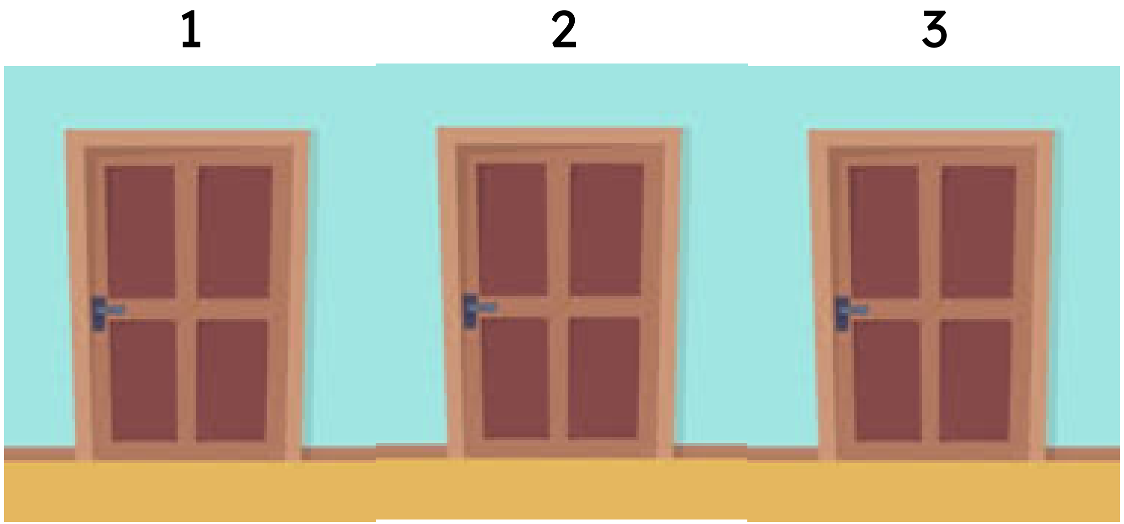

1
2
3
⅓
⅔

Scenario A: you picked the right door to start with and I just show you one of the other incorrect doors, in which case staying will make you win.

Scenario B: you chose the wrong door and I show you the other incorrect answer, in which case switching will make you win.

For A to happen, you need to have chosen the winning door to start with ($\frac{1}{3}$ chance) and for B you need to have chosen the wrong door to start with ($\frac{2}{3}$ chance)

Therefore A only happens 1 in 3 times, and B happens 2 in 3 times, so you’re more likely to win when you switch.



Door the car is behind	Door you initially chose		
	1	2	3
1	STAY	SWITCH	SWITCH
2	SWITCH	STAY	SWITCH
3	SWITCH	SWITCH	STAY

In order to win you need to switch 6 out of 9 times - ⅔

When the host opens Door 3, the probability of a car being behind Door 2 increases from ⅓ to ⅔ - so it's actually better to switch from your original choice of Door 1 to Door 2!

(Congratulations, you've won a car!)

How many people need to be in a room before the probability 2 of them share a birthday surpasses 50%? Well the answer is 23 (50.73% chance) - much lower than most of us initially think. Using a mixture of probability and combinatorics, we can deduce the answer. The Birthday Problem is an example of a veridical paradox, which is a situation that seems counterintuitive or even impossible until you break it down. Another example is the Monty Hall Problem, which proves that on a gameshow with 3 doors where 1 door contains a prize, you are better off switching from your original choice if the game show host opens one of the other doors with no prize behind it. The probability you win if you do is $2/3$ - double what it is if you stick with your first choice.