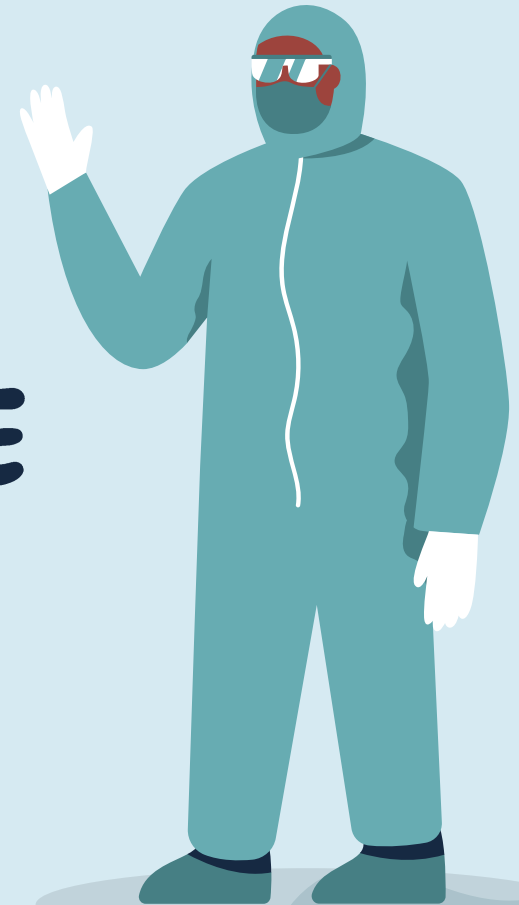


MATHS IN MEDICINE

DISEASE MODELLING AND EPIDEMIOLOGY



WHAT IS MATHEMATICAL MODELLING?



Models describe our beliefs about how the world functions. In mathematical modelling, we translate those beliefs into the language of mathematics.


This has many advantages:

- Mathematics is a very precise language
- Mathematics is a concise language, with well-defined rules for manipulations.
- We have the results that mathematicians have proved over hundreds of years at our disposal
- Computers can be used to perform numerical calculations

EXAMPLES OF MATHEMATICAL MODELS

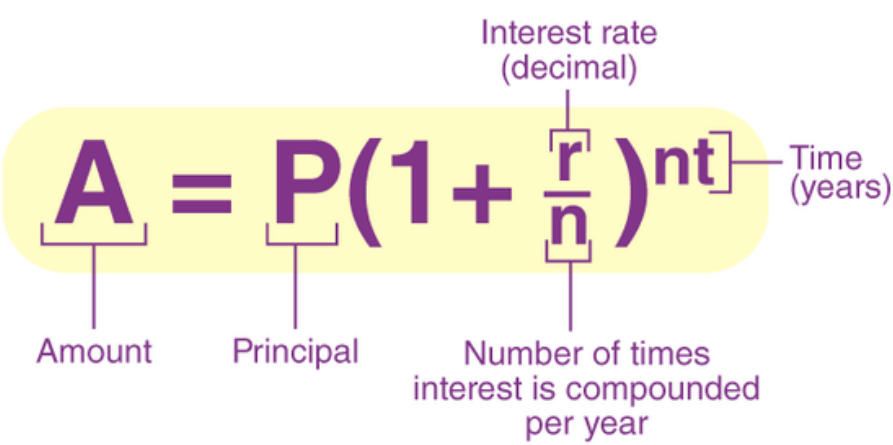
Compound Interest

This models the growth of an investment over time

COMPOUND INTEREST FORMULA 

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$

Amount Principal Interest rate (decimal) Time (years)
Number of times interest is compounded per year



EXAMPLES OF MATHEMATICAL MODELS

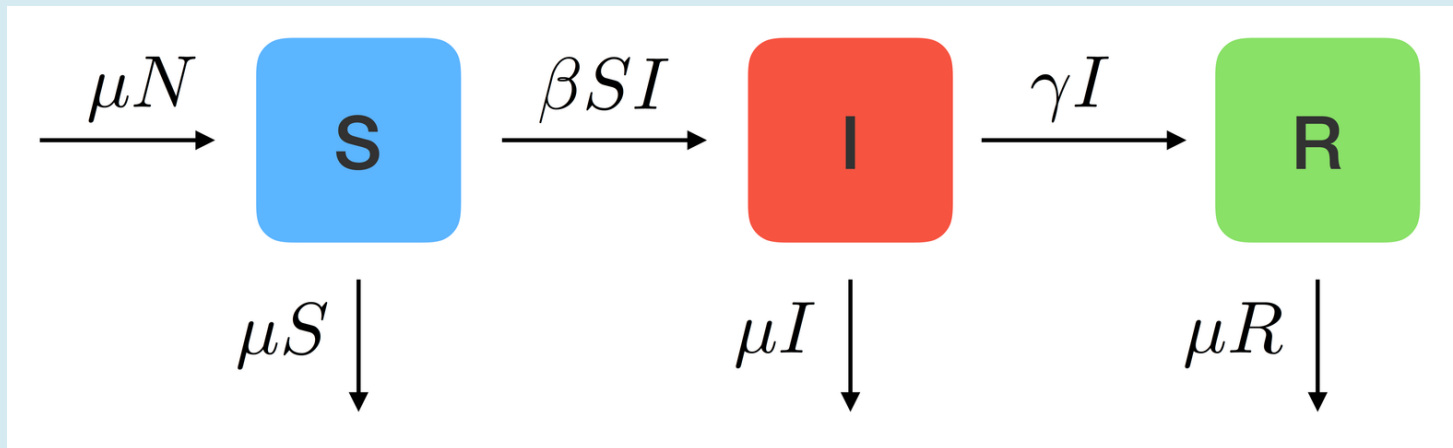
The Wind Chill Factor (2001)

This calculates how cold air feels on human skin by modelling heat loss from a face in the wind

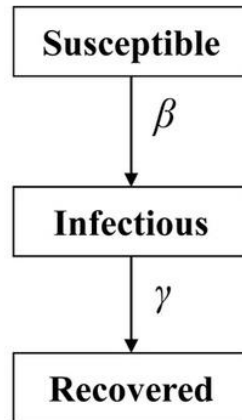
The Wind Chill Formula!

$$35.74 + 0.6215T - 35.75v^{0.16} + 0.4275Tv^{0.16}$$

THE SIR MODEL!!!



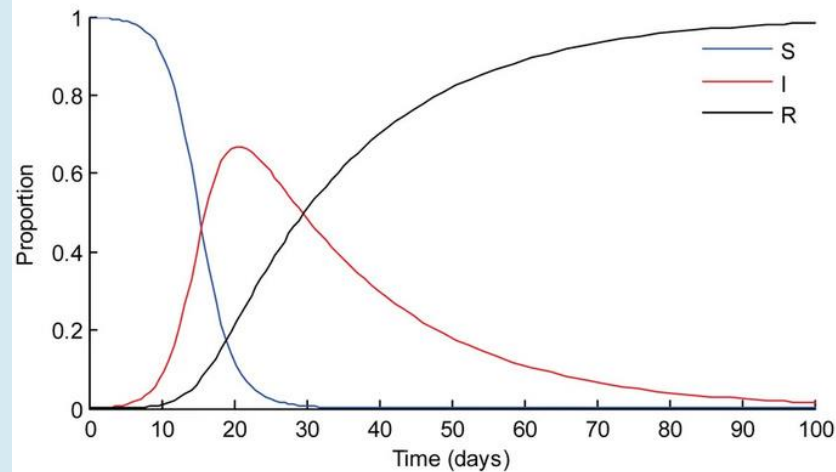
THE SIR MODEL!!!



$$\frac{dS}{dt} = -\beta SI$$

$$\frac{dI}{dt} = \beta SI - \gamma I$$

$$\frac{dR}{dt} = \gamma I$$



WHY DO WE USE A MATHEMATICAL MODEL?



- Mathematical modelling is a powerful and increasingly indispensable tool in public health, widely used to predict, analyse and control the spread of infectious diseases.
- These models simulate the transmission dynamics of diseases within populations, allowing public health officials to understand potential outbreak scenarios
- The World Health Organization (WHO) recognizes the critical role of modelling in enhancing global health security

WHAT IS THE SIR MODEL?

SIR (Susceptible, Infectious, Recovered)



The SIR model is one of the most basic models for describing the temporal dynamics of an infectious disease in a population

It compartmentalises people into one of three categories:

Susceptible (S)

Infectious (I)

Recovered (R)

The SIR model describes the number/proportion of people in each compartment at every point in time

UNPACKING THE SIR MODEL



B, ST AND N

Every day, a contagious person will randomly move around the population and infect any susceptible person they come into close contact with.

We represent the average people each day that come into close contact with the infected person with β (beta)

The SIR model states that if there are S_t susceptible people on day t , the expected number infected by our carrier is $\beta S_t / N$ where S_t / N is the probability a close contact is with a susceptible person.





Y, B, R0

The infectious carrier will continue this process of randomly contacting people each day until they are no longer contagious

The SIR model represents the probability that a contagious person becomes non contagious as γ (gamma)

Therefore, the average number of days that someone stays contagious is $1/\gamma$

When γ and B are combined, they make R0 which is known as the basic reproductive number. Specifically, γ/B is equal to R0.



$$R_0 = B/Y$$

If R_0 is equal to B/y then this represents the expected number of close contacts a day multiplied by the average of days contagious ($1/y$)

But why does this matter?

It matters because if B is greater than y (R_0 is greater than 1), we will likely have an epidemic on our hands as each infected person will infect more than one other on average. They will then infect more than one other and so on.

This leads to the exponential growth of a disease spreading.



WHAT ABOUT THE EQUATIONS?

The equations keep track of how fast **changes** happen in how many healthy people become infected and how many infected people recover.

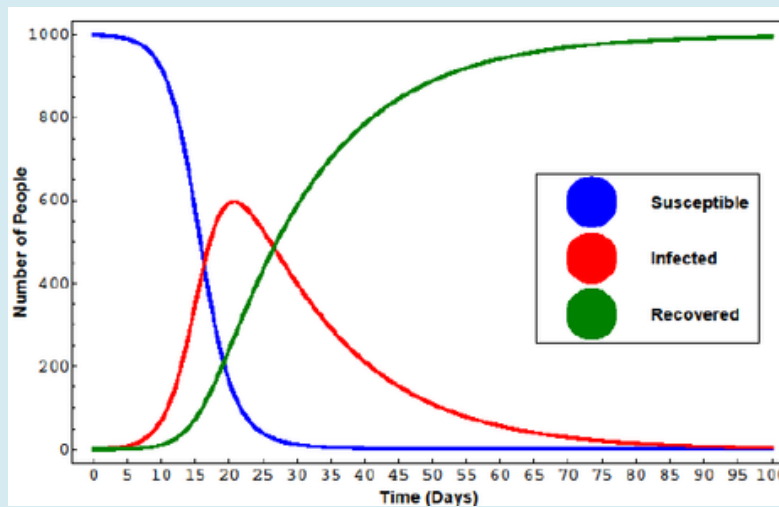
$$\frac{dS}{dt} = -\beta SI$$
$$\frac{dI}{dt} = \beta SI - \gamma I$$
$$\frac{dR}{dt} = \gamma I$$



$$dS/dt = -\beta SI$$

$dS/dt = -\beta SI$ essentially means that S (the number of susceptible people) decrease over time because susceptible people meet infected people

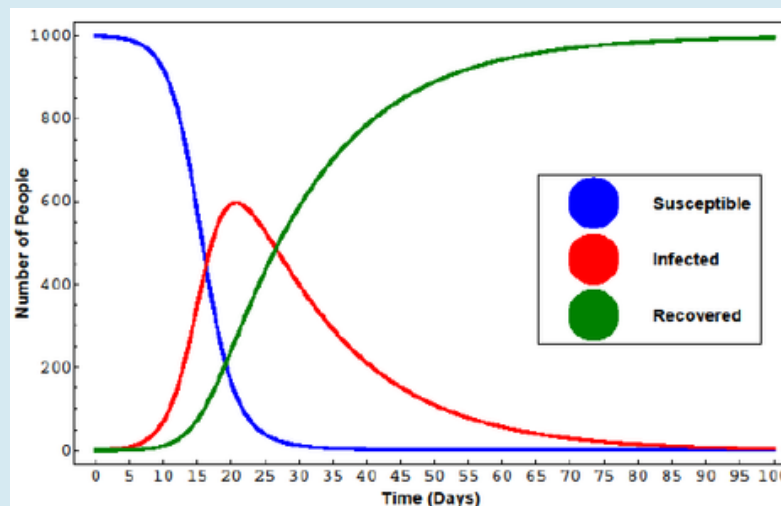
When B is big, it means the disease spreads fast. If B is small in this equation, the disease spreads slowly.



$$DI/DT = BSI - YI$$

This equation means that the infected people increase when healthy people catch the disease and decrease when sick people recover

When y is large, people recover quickly
When y is small, people stay sick longer

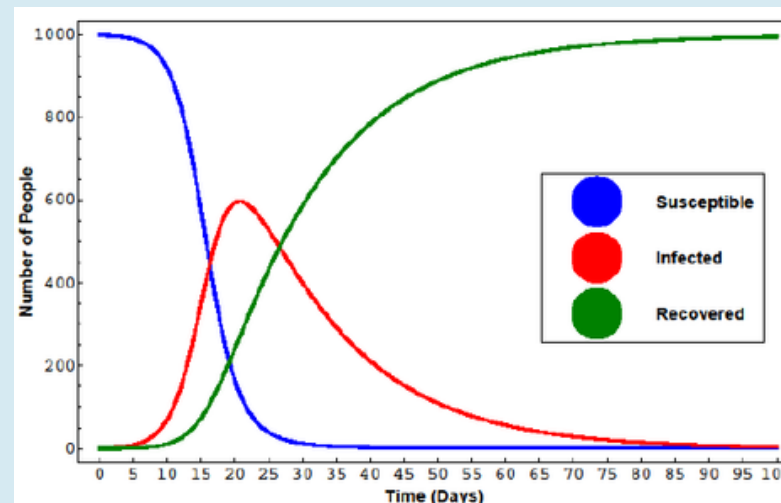


$$DR/DT = YI$$

Recovered people grow when infected people get better.

yay!

This is usually represented on the curve as a steady rise.



WHY DO DISEASES EVENTUALLY STOP SPREADING?

At the beginning of an outbreak, almost everyone is susceptible so BS is large. Therefore, on average, each infected person creates more than one new infection on average.

As the epidemic progresses, the susceptible people are depleted and S steadily decreases. This causes BSI to weaken yet recovery continues at the same rate (γI).

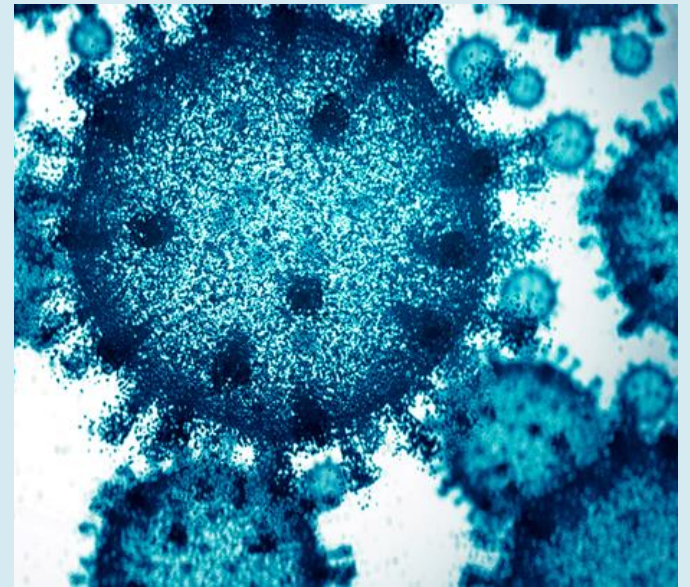
At a point, BS will be equal to γ , this is when the epidemic reaches its peak because new infections equal recoveries.



THE SIR MODEL AND COVID-19

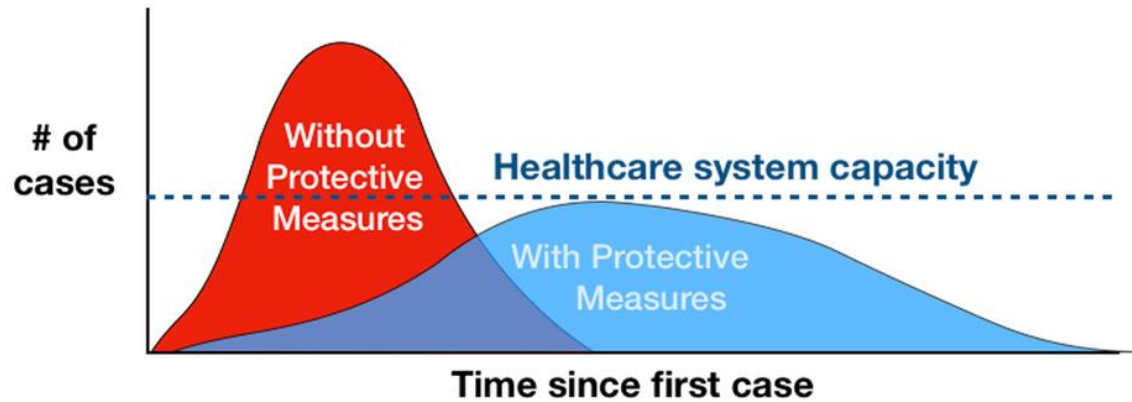
It is easier to fully appreciate how useful the SIR model is by seeing how it was used during the COVID-19 pandemic

- Estimating how fast the virus would spread by calculating R_0
- Forecasting hospital demand such as the number of ICU beds needed and ventilator demand
- Evaluating interventions such as lockdowns and social distancing
- Planning vaccination strategies
- “Flattening the curve”



"FLATTENING THE CURVE"

THE

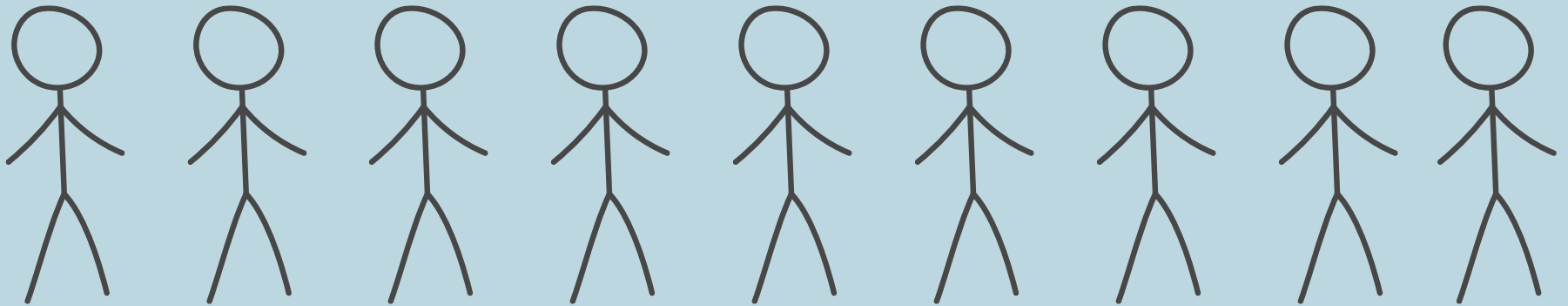


Adapted from CDC / The Economist



EXAMPLE!!

A disease called Dropping Maths Syndrome (DMS) has broken out...



Elyza, Eve, Ella, Daisy, Miley, Amelie, Ms Olatunji, Ms Shannon, Ms Darwin

Population: 9 people
Transmission rate of DMS: 0.6
Recovery rate of DMS: 0.2

DAY 1...



Elyza becomes infected with DMS.

S: 8

I: 1

R: 0



DAY 1...

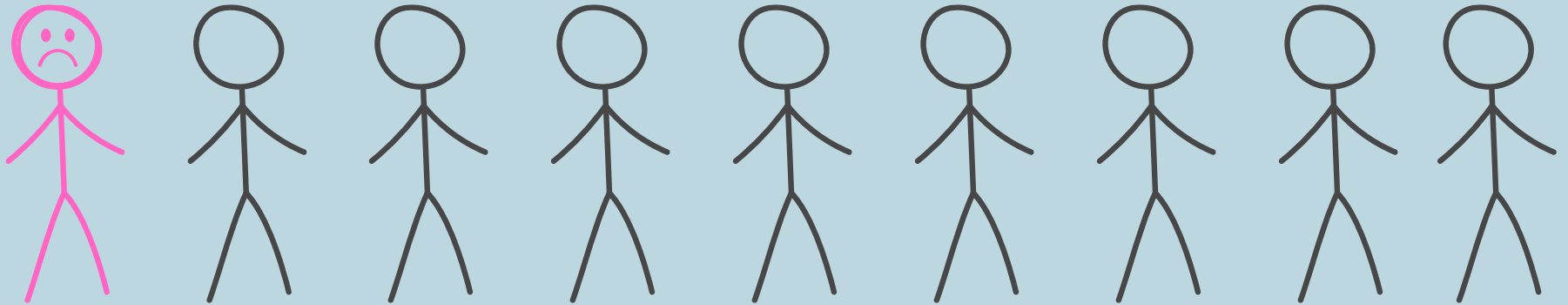


To calculate early epidemic growth (how fast the number of people with DMS is changing), we use dl/dt which is equal to $BSI - \gamma I$

If it is positive, infections are increasing
If it is negative, infections are decreasing
If 0, the epidemic is at its peak



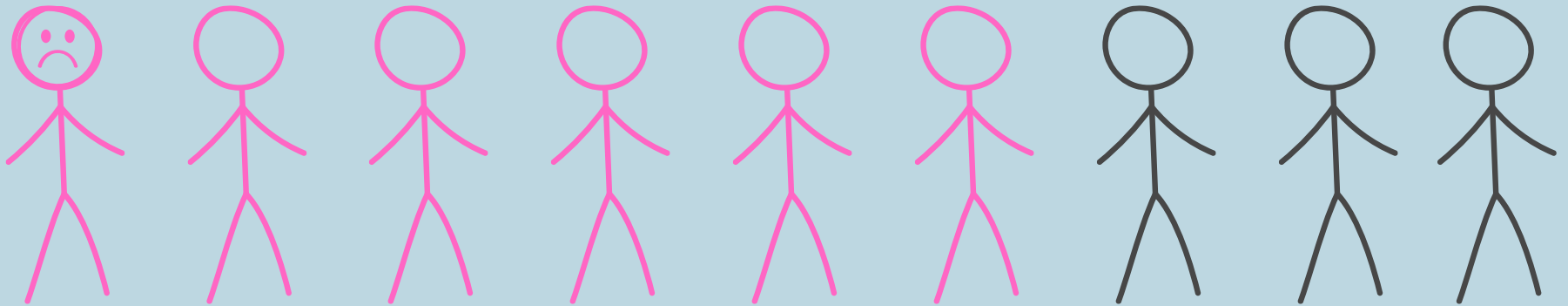
DAY 1...



Therefore, the infected population is increasing at a rate of about 0.33 of a person per unit time.

As 0.33 is greater than 0, the infections are growing overall.

DAY 7



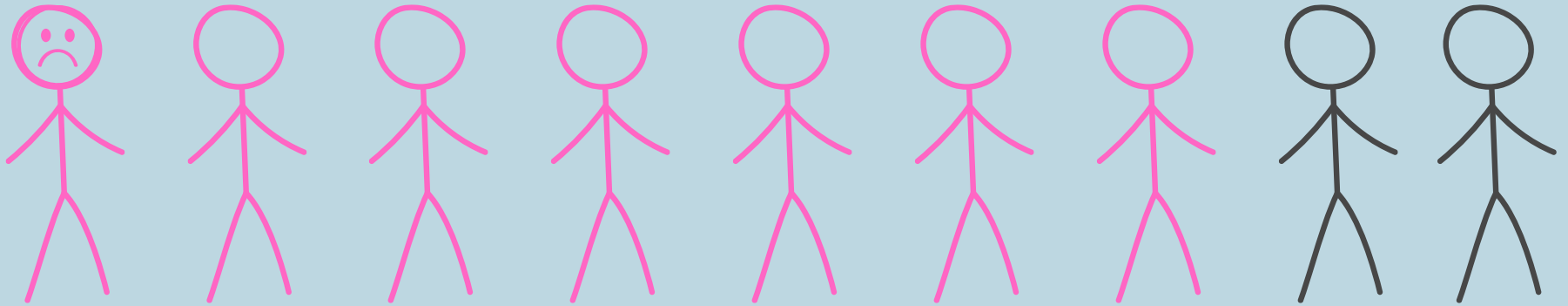
From what we've calculated, the epidemic peaks exactly when 3 susceptible people remain.

yay!

As new infections = recoveries, active infections stop increasing.



DAY 8



If susceptible people fall below 3, recoveries outpace new infections and the epidemic of DMS declines!!!
(exponentially)

yay!

This shows how the epidemic of DMS mathematically creates the conditions for its own decline.
This is the central mechanism of the SIR model.



**THANK YOU
FOR LISTENING**



REFERENCES

https://people.maths.bris.ac.uk/~madjl/course_text.pdf

<https://pmc.ncbi.nlm.nih.gov/articles/PMC8993010/>

<https://people.wku.edu/lily.popova.zhuhadar/>




150 WORD SUMMARY



My research focused on the role of mathematics in medicine, specifically in disease modelling and epidemiology. In this presentation, I explained why mathematical models are important, as they help scientists and healthcare professionals understand how diseases spread, predict future outbreaks, and make informed public health decisions. The main model I discussed was the SIR model, which divides a population into three compartments:

Susceptible (S), Infectious (I), and Recovered (R). I described how individuals move between these compartments as a disease spreads through a population. I also explained the notation used in the model such as the ones that represent transmission (β) and recovery rates (γ). I discussed how the SIR model was used during the COVID-19 pandemic to estimate infection levels, predict healthcare demand, and evaluate the impact of interventions such as lockdowns and social distancing. This demonstrated the practical value of mathematics in medicine, which is what



I want to study in the future. Finally, I created my own example to demonstrate how to use the numbers in the differential equation dl/dt .